Philippe Colliard
Who I am

Pour l'épisode en français, cliquez ici ⇨ Les maths comme je les aime … et comme je les raconte ! 🙂

All episodes published : https://www.mathasilikeit.com

Math as I like it /5

What if numbers were just names? 🙄

*(Episodes 5 to 9 are very loosely based on the article "Les harpes de Thalès" (The Harps of Thales) that I published on the website "Images des mathématiques" (CNRS). Very loosely because here I don't write math, I merely touch it: **I am telling it... as I like it** 😊 – and as I would like to share it with you. But of course, fundamentally it is the same math).*

I've heard your sighs and murmurs:

*– you're... um, tiring us out with your geometry, your points, your points, your points! We want **real** math, math with numbers everywhere!*

Okay, fine, let's tackle numbers. All numbers, from integers to complex numbers. And the numerical structures that go with them. Obviously, it's going to take a little time and a few episodes. But who's in a hurry?

We're going to hunt down numbers together, bringing them out into the light little by little, even though all they want is to be left alone, each in their own point.

*– In their **what?** You mean "in their **place**"?*

Uh, yes, yes, of course! In their place! So we're going to hunt them down. Note that I didn't say "create them"; they've existed for eternity, a bit like – if you remember *my first episode* – the points that have been there for eternity, waiting for a point object to come and visit them one day. Well, if points wait?

– Oh no! Don't start with your points again!

I knew I was going to upset you! But I can't help it, because it's precisely through points that I intend to approach these numbers, that *we*'re going to approach them! It's not the usual method, so what? I really like it... and it's so visual!

– Hmm... and what does your unusual approach involve?

That observing a number is actually just observing a point in a different light. Or, if you prefer, that each point carries its number with it.

Or that it's all a question of glasses: you look at an object, through certain glasses you see a point, while through others you see a number. If by chance you've read "*the divisors of a number, from its spectrum*" (also in *Images of Mathematics*) it might ring a bell?

– But what you're saying is silly: you can't add or multiply points! What about the whole construction of numerical structures?

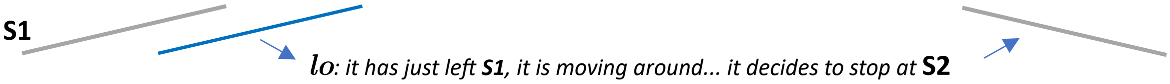
Of course we can add or multiply points. And numerical structures, well, they already exist... among points.

Okay, it's clear that if I don't dot my i's and cross my t's, it's not going to work. So let's give it a try, shall we? **Today, you'll join me in a quick overview of a *geometric* construction of integers – *all* integers**... and if I've convinced you, you might want to read the following episodes (or maybe even "Thales' harps" on Image des Mathématiques, but that won't be just a quick overview!).

You'll see, it won't take long because in the previous episodes we've already set up almost everything we need... All we're missing is the notion of "congruent segments," defined by Hilbert in his axiomatic system. Don't panic, it's a big name for a very simple idea:

you observe a segment **S1** (it's part of a line, so it's a place: you can't move it!)
you occupy this segment with a linear object lo: this is an object (a hyper-thin wire), it can move!

You move lo, and it occupies (elsewhere) a new segment **S2**:



And that's it: **S1** and **S2** are two congruent segments😊

Yes, it seems like a complicated way of saying that they are "equal." But in math, "equal" has a very specific meaning: "**S1** equals **S2**" would mean that **S1** and **S2** are two different names for the same object (object in the mathematical sense, i.e., pretty much anything that is sufficiently defined: a place is an object!)... but **S1** and **S2** are *not* the same object: they are two different places! So these two segments are *not* "equal"!

Of course, I could have said "**S1** and **S2** are two segments that have the same length" – but length is a measurement, and measurements use numbers... and we haven't figured out those numbers yet!
Would you like some aspirin?

Are you ready? Here we go: we know a lot about points, lines, segments (congruent or not)... but we've *never* heard of numbers.

Now ***imagine*** (because of course it's all in your head!):

you are immersed in a geometric universe, a universe of points.
You choose two points in this universe and call them **A** and **B**. No, you can't see them; points are just places.

Imagine the line that contains ("passes through") these two points: call it ***d***. No, you can't see it either, a line is also just a place. But since it is extremely difficult to focus your mind on invisible objects, you will do as everyone else does: you will *pretend* to see **A**, **B**, and ***d***, and to help you, I will represent them with colored dots and lines on a white background (as I did for S1 and S2):

However, since you are imagining, go even further and imagine that your geometric universe is a black void that you have the power to illuminate, "lighting up" the points you choose by injecting them with bright punctual objects (objects that you can then choose to turn on or off).

And let's agree on another principle: in this black void, I will represent the "lit" points in red and the other elements in any other color – they are only there to help our imagination, to guide it (when we look at the Big Dipper, we only see the stars, don't we?).

And now, I light up points **A** and **B**:

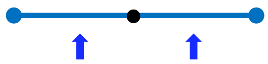
Ideally, you should imagine yourself facing a black immensity from which two lit points emerge... these will be the first two stars in the universe of whole points – and whole numbers. The other points will light up soon😊 !

I decide to call **A** "**the whole point origin of *d***" and to name **B** "**the successor of A on *d***"
...and therefore, logically, **A "the predecessor of B on *d*"**!

YES: I could have decided that the whole point origin of *d* would be B and that A would be its successor... it's just my choice.

There you go, you've done the hardest part. I suggest letting AI take over with a little ***surveyor robot,*** nothing too complicated:

at **the heart of *the surveyor*** is this double segment:

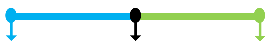


Each of the two "small" segments is congruent to the segment [AB]

(another twisted way of saying that the black dot is the midpoint of the segment... but the concept of midpoint is based on that of length – and therefore numbers, which we still don't know 😊)

... then a two-color linear segment *object*, constructed from this segment and equipped with three sights. But unlike the segment, it can move:

The two-color *surveyor* *(black is not a color 😊)* →



← and its three sights

... and finally, a program that is not too complicated:

- *the surveyor* moves at will along a line parallel to *d*:
- when two of his sights indicate two lit points on *d*,
  it lights up (if it hasn't already done so) the point on *d* indicated by the third sight:

  if this third sight is the blue one,
  we will say that the point it indicates is the predecessor of the one indicated by the black sight,

  if this third sight is green,
  we will say that the point it indicates is the successor of the one indicated by the black sight

  (... and if this third viewfinder is black, we will say nothing: the point it indicates is already lit 😊)
- it must light up at least two new points every day, one on one side of A, the other on the other side... but that's just to make him work a little!

And that's it! *(It doesn't seem like much, does it? Yet we have just set up the whole scenario of "whole points"... and their numbers!)*

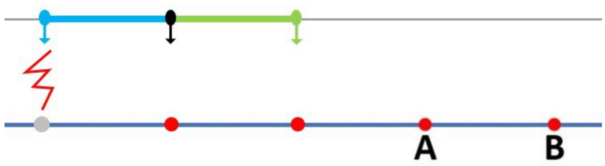
But ***why*** does it follow this program? It has no idea, of course. If it had read The Little Prince – and if it could talk – it would tell us "it's the instructions," like a distant human ancestor of it, a lamplighter by trade.

In fact, perhaps it has heard of this ancestor, perhaps it's sorry for it, it who is not subject to any schedule, any rhythm, (almost) any progression even: it can move forward or backward at any time as far as it wants, or do nothing, just wonder, for example, if its work will ever end (and hope that it won't: it gives it a lot of time to think).

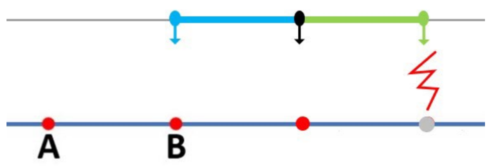
(Forward? Backward? There is no direction on a line, but since I have decided to call B the "successor of A on d" and since "succeeding to" often makes us think of "moving forward in time," we can decide that for the surveyor, "forward" means "toward the successors")

Then, of course, we let *the surveyor* work *(here, he has already lit 3 points, he lights 2 new ones)*:

on one side of A...



...on the other side 😊



And here are our "whole points"... well, some of them:

(Imagine them on a black background, you no longer see A or B or d, just a huge ribbon of stars in your geometric universe!)

– Okay, okay, that's all very nice, but where are the numbers in all this? ***The numbers???***

Why are you getting so worked up? I'm getting to the numbers!

Let's go back to our ribbon of whole points, this time with a few indications in black: A, B, and the names I give to two other points, T and Y:

(Why these points? I'm not sure: on a map, they could be the houses of two friends along a road?)

Now imagine that other people have this map in front of them, with only points A and B already marked. They can't see me, so how can I describe to these people in writing the points I have labeled T and Y?

I could, of course, use letters or even words, for example: "Write the word ABSOLUTELY from AB... and stop at T: AbsoluT... then finish the word backwards starting from A: YleA."
But that's a bit convoluted, isn't it? It's like a treasure hunt!

I could also, which would be more reasonable, write to them:
"Go from A to B, then move forward one point, and then another, and another, and another, and another. Stop! You've reached T."

But if T were a point really far from A and B, can you imagine how many pages that would take?

I could also symbolize the successive segments needed to reach T or Y **from A**, for example like this:

A – B – – – – – T   and for Y: Y – – – A –B

And if I want to save some space, let these other people know that **I'm *always* starting from A**, replace the "A – B" block with an arrow to indicate whether I'm going from A to B or in the other way, and no longer write T or Y:

for T: → – – – – –   and for Y: ← – – –

Or even decide that if I'm going "forward," I won't mark anything:

for T: – – – – –   and for Y: ← – – –

Well, I won't insist, you understand the principle:

by deciding to symbolize the segments with vertical lines (it takes up less space), we quickly arrive at something that foreshadows Roman numerals: for T , | | | | | and   for Y ,   ← | | |

(For Arabic writing, in texts 5a and 5b, I present two "stories by Father Colliard":
"Hi-Ati, the creation of digits", then "Hi-Ati, the creation of tens"... two completely false stories😊 !)

In short, after centuries of wandering, a large part of the world finally agreed on something like this:

Whole numbers would therefore be nothing more than labels attached to whole points on a line, code names for these points, constructed using a logical algorithm that allows them to be quickly located in relation to the two points chosen to construct this sequence of whole points on the line. To *identify* them.

***   ***   ***

This is just a story, as you well know: it wasn't by following the path of this article that whole numbers were discovered, and if I have been able to present this approach to you, it is only because great minds worked long and hard to transform what was a jungle full of brambles into a peaceful garden.

But why might this approach be interesting? There are two main reasons:

***it does not single out any point on the line** and **it is global**:*
(To paraphrase George Orwell: all points on a line are equal, there is no reason why "some should be more equal than others"!

***It does not single out any point on the line** in the sense that we do not start from an exceptional point:*

> *a line has no starting point or preferred direction: points A and B that I chose could just as easily have been two other successive points on my ribbon of whole points without changing the ribbon – just the numerical identification of its points.*
>
> *(And if – but that would take us too far afield – I changed the "gap" between my starting point and its successor, I would of course no longer get the same ribbon of whole points, but by moving closer to or further away from the line (by "zooming in"), I would find the same visual distribution.)*

***It is global**:*

> *From the outset, I observe the line as a whole: there is no artificial separation between whole points whose essence would be different – positive or negative: a point is a point!*
>
> *And as you will see (possibly😊) in the following episodes, there will also be no artificial separation between whole, rational, and real points (and numbers):*
>
> *because these will be geometric operations – between points on the line – and geometric demonstrations of their properties, the operations that I will introduce shortly will be designed from the outset to apply under the same conditions to all points on the line (whether we have already "found" them or not)... then very simply transposed to the numbers associated with these points.*
>
> *(I will therefore not need to "invent" operations specific to natural integers and then think about how to extend them to relative integers, then to rational numbers, then to real numbers).*